

On the performance
of unitary models
of household labor supply
estimated on “collective” data
with taxation

Denis BENINGER
François LAISNEY

Denis BENINGER *, François LAISNEY **

Performance des modèles unitaires d'offre de travail des ménages estimés sur la base de données « collectives » en présence de taxation

Résumé – Cet article compare les modèles unitaire et collectif sur la base de données collectives simulées en présence de divers types de taxation des revenus. Nous distinguons en particulier un système d'imposition individuelle et un système d'imposition jointe. En spécifiant un modèle unitaire flexible, du type de ceux utilisés dans la littérature empirique, nous obtenons des estimations assez différentes selon le régime fiscal utilisé pour simuler les données. Nous trouvons aussi des différences substantielles dans les ajustements de l'offre de travail prédits à la suite d'un changement de régime fiscal. Nos résultats illustrent également le fait que le calibrage des paramètres d'une réforme fiscale peut être lui-même fortement affecté par l'utilisation d'un modèle unitaire lorsque les données sont générées par un modèle collectif. Enfin, nous discutons les distorsions qui affectent l'analyse de réformes fiscales en termes de bien-être dans une telle situation. Les résultats suggèrent que des efforts accrus devraient être consacrés à la recherche sur l'estimation de modèles collectifs dans les situations complexes auxquelles on est confronté lors de l'analyse de réformes fiscales. Ceci inclut des contraintes budgétaires donnant naissance à des ensembles de budget non convexes, ainsi que la modélisation de solutions en coin pour les offres de travail des conjoints.

Mots-clés : allocations Pareto optimales, évaluation de réformes fiscales, données simulées

On the performance of unitary models of household labor supply estimated on “collective” data with taxation

Summary – This paper compares collective and unitary models on the basis of simulated collective data with income taxation. We distinguish the cases of individual and joint taxation. Estimating a flexible unitary model, we obtain strikingly different “preference” parameters depending on the type of taxation. We also obtain substantial differences between predicted adjustments to labor supply following a switch between tax regimes. Our results show that even the design of revenue-neutral reforms may be heavily distorted by the use of a unitary model on collective data. Finally, we discuss distortions affecting the welfare analysis of reforms on the basis of unitary estimates when the model generating the data is a collective model. The results suggest that increased efforts should be devoted to the estimation of collective models with taxation.

Key-words: Pareto optimal allocations, policy evaluation, simulated data

* Zentrum für Europäische Wirtschaftsforschung (ZEW), L7 1, D-68161 Mannheim, Germany
e-mail: Beninger@zew.de

** BETA-THEME, Université Louis Pasteur, 61 avenue de la Forêt Noire, 67000 Strasbourg, and ZEW
e-mail: fla@cournot.u-strasbg.fr

We are grateful to Katherin Barg, Pierre-André Chiappori, Bertrand Koebel, Isabelle Maret, Hubert Stahn, Frederic Vermeulen and Elke Wolf for careful reading of a previous version. Thanks also to two anonymous referees, whose comments helped us to revise the paper. All errors remain our own.

COLLECTIVE MODELS of household consumption and labor supply behavior, introduced by Chiappori (1988, 1992) and Apps and Rees (1988), allow the representation of individual behavior inside the household. By contrast, the more widely used unitary models consider household behavior as resulting from the decisions of a single individual, occulting the fact that most households are composed of several individuals who take part in the decision processes. This precludes the analysis of intra-family redistribution of household resources. Unitary models treat the family as a black box, so that the income distribution inside the family cannot be reconstructed (see *e.g.* Lechene, 1993). The main difficulty with the unitary models concerns the aggregation of preferences (see Arrow, 1951 and Hildenbrand, 1994) and of individual demands (see Deaton and Muellbauer, 1980). Although Samuelson's consensus idea (1956) and Becker's altruistic model (1991) generate consensual behavior as regards the resource allocation within the family, the underlying assumptions have been criticized as highly restrictive (see *e.g.* Ben Porath, 1982 and Bergstrom, 1989); indeed allocations within the household may well be conflictual (see Sen, 1984).

For a model of allocation of leisure and consumption of an aggregate good, Chiappori (1988, 1992) shows that his definition of collective rationality, which imposes Pareto efficient allocations, implies a set of restrictions on the labor supply functions, and that these are sufficient to identify the individual preferences and the income sharing rule up to an additive constant. These conditions consist in a set of restrictions on the partial differentials of the labor supply functions.

The basic model has been extended in several directions, including household production and children (Apps and Rees, 1996, 1997, 1999; Chiappori, 1997; Chiuri, 1999; Bourguignon, 1999), and the presence of more than two decision makers in the household (Browning and Chiappori, 1998). Empirically relevant discrete decisions, such as non-participation of one of the household's members, introduce further difficulties. Even in that case, complete identification (up to an additive constant) has been shown for collective models with a linear budget constraint (Blundell *et al.*, 1998) and with a convex budget set (Donni, 2003; Beninger, 2003). More details concerning the literature on collective models can be found in the excellent survey of Vermeulen (2002).

Yet vast research efforts remain necessary for the implementation of collective models in all situations where unitary models are used in practice, in particular in the presence of non convex budget constraints, and for modelling intertemporal allocations. The aim of the present study is to assess the potential returns of such a research program: By simulating data from the collective model we circumvent the difficulties that are still connected with its estimation, but we are able to point out the many distortions created by the use of a unitary model.

A more pointed way to describe our aim is to say that we set out to exhibit a counter-example to the claim "*For practical purposes, the choice between the collective and unitary representations of household decision processes does not matter much*". While the literature – especially in mathematics – abounds with construed and mind-boggling counter-examples, a main message of this paper may be that no such contortion is

needed in the present context. We very straightforwardly elaborated a simple model, realistic within this simplicity, but on the other hand unrealistic on many accounts (we consider only childless couples, with no systematic difference between the spouses other than the fact that their wage rates are drawn from different distributions), and directly found large distortions. A further development of this study, focusing on realistic calibration for six European countries, taking into account the existing fiscal systems and reforms under discussion, as well as the structure of the population, and investigating how important the distortions illustrated here may be for these different countries, is documented in a set of four papers – Bargain *et al.* (2006), Vermeulen *et al.* (2006), Myck *et al.* (2006) and Beninger *et al.* (2006) –, thus providing a large scale sensitivity analysis¹. Yet, as a referee pointed out, in the real world there are many reasons for a unitary model to be misspecified, on top of the non-respect of the collective nature of household behavior. One of them relates to dynamics, another one to the way in which households approximate the role of taxation and transfers in their choices. Estimated preferences account for that limited rationality and their parameters counteract, to some extent, the full account of taxation imposed by the model. Here, behavior is static and incorporates the tax system fully: The use of artificial data allows to rule out these other sources of misspecification and improves the assessment of the unitary model. Admittedly, another research goal could be to theoretically characterize the sets of preferences and bargaining rules for which distortions do or do not occur, but it is beyond our present purpose.

However, the main reason why we do not perform the symmetric exercise of simulating unitary data and analyzing distortions from the use of collective models is: The unitary model is regularly rejected in the empirical literature².

We focus on the analysis of tax reforms because this type of exercise often motivates the estimation of household preference parameters. Our results show that even the **design** of revenue neutral reforms may be heavily distorted by the use of a unitary model on collective data.

In the following section, we briefly present the two models. In the third section, we explain how the simulated data sets are generated. These are described in the fourth section. The econometric specification of the estimated unitary model, which draws on van Soest (1995) is discussed in the fifth section. Estimation and policy simulation results are discussed in the sixth section, with four subsections presenting the estimates, their use in predicting labor supplies, and the positive and normative analysis of tax reforms, respectively.

¹ These papers, published in a special issue of the *Review of the Economics of the Household*, are hereafter referred to as the REHO papers.

² See Browning and Chiappori (1998) and Vermeulen (2002) for several references.

The representation of household behavior

In this section we briefly present the unitary and collective models of allocation of leisure and consumption of an aggregate good at the household level. For simplicity, we focus on childless married couples.

Unitary models

Unitary models assume that the household maximizes a unique utility function, independent of prices and incomes. They also assume the pooling of family incomes. Thus, for these models, the allocations are deduced from the maximization of a utility function under budget and time constraints:

$$\begin{cases} \max_{c_f, l_f, c_m, l_m} U(c_f, l_f, c_m, l_m), \\ c_f + c_m \leq g(l_f, l_m, w_f, w_m, y), \\ 0 \leq T - l_i \leq H \quad \forall i = f, m, \end{cases} \quad (1)$$

where c_i , l_i , w_i , y , H and T represent agent i 's ($i = f, m$, for wife and husband, respectively) consumption, leisure demand and gross wage rate, the couple's unearned income, the maximum work time and the total time in a week. The maximum number of hours of work was introduced for numerical reasons, but it could be justified by the existence of legal constraints. Function $g(\cdot)$, which describes the tax system, is assumed to generate convex budget sets. While this restriction can easily be relaxed for the unitary model, this is not (yet) the case for the collective model. The utility function $U(\cdot)$ is increasing in its arguments and quasi-concave.

With the unitary model, the intrahousehold distribution of resources plays no role. Yet the question of intra-family redistribution of incomes can be crucial in determining household choices (see *e.g.* Lundberg *et al.*, 1996) and answers are important if institutions want to conduct efficient and fair economic and social policies (see *e.g.* Haddad and Kanbur, 1990). Collective models can offer such answers.

Collective models

In these models, consumption choices are derived by assuming only Pareto optimality of allocations, and this single *a priori* assumption defines the collective rationality concept used. Following Chiappori (1988), we focus on the case where individuals' preferences are egoistic (each spouse's utility is only defined on own consumption and leisure). In that case, and for a linear household budget restriction, household behavior can be represented sequentially, using an explicit sharing rule for the unearned income. Chiappori shows that individual preferences and the sharing rule are identified from observable behavior, up to a constant. The extension to the

case of non linear, but convex, budget sets has been introduced by Donni (2003): For egoistic individuals the individual preferences and the unearned income sharing rule can still be identified up to an additive constant (see also Beninger, 2003).

In the case of egoistic preferences, collective models with convex budget sets have the following **centralized** representation:

$$\begin{cases} \max_{c_f, l_f, c_m, l_m} U^f(c_f, l_f) + \lambda(w_f, w_m, y) U^m(c_m, l_m), \\ c_f + c_m \leq g(l_f, l_m, w_f, w_m, y), \\ 0 \leq T - l_i \leq H \quad \forall i = f, m. \end{cases} \quad (2)$$

If the relative weight of the husband, λ , is constant, model (2) is a special case of a unitary model, with separability in the partition $(c_f, l_f), (c_m, l_m)$, a point to which we shall return in the fifth section. Alternatively, the **sequential** representation is ³:

$$\begin{cases} \max_{c_i, l_i} U^i(c_i, l_i), \\ c_i \leq \tilde{w}_i(T - l_i) + \tilde{y}_i, \\ 0 \leq T - l_i \leq H \quad \forall i = f, m, \end{cases} \quad (3)$$

where \tilde{w}_i is i 's implicit (or shadow) wage at the optimum $(c_f^*, l_f^*, c_m^*, l_m^*)$:

$$\tilde{w}_i = \frac{\partial g}{\partial l_i}(l_f^*, l_m^*, w_f, w_m, y). \quad (4)$$

If we define the household's implicit unearned income, \tilde{y} , as:

$$\tilde{y} = g(l_f^*, l_m^*, w_f, w_m, y) - \sum_{i=f, m} \tilde{w}_i(T - l_i^*), \quad (5)$$

there exists an implicit income sharing rule which determines \tilde{y}_f as $\tilde{y}_f = \tilde{Y}(\tilde{w}_f, \tilde{w}_m, \tilde{y})$, such that the sequential representation (3) is equivalent to the centralized form (4) of the model. The equivalence can be shown with the implicit functions theorem, and the details are given by Beninger (2003), and Donni (2003).

³ See Chiappori (1992) and Apps and Rees (1988), for the linear case, and Donni (2003), for the convex case.

In our simulations, the implicit income sharing rule $\tilde{\Upsilon}(\cdot)$ will be related to the effective sharing rule which the spouses use to share gross unearned income. This sharing rule is defined as

$$\Upsilon: (w_f, w_m, y) \mapsto y_f/y, \quad (6)$$

where y_f denotes the wife's share of gross unearned income.

Simulation of collective data

Data were generated from the following specification of the sequential representation (3) of the collective model:

$$\begin{cases} \max_{c_i, l_i} U^i(c_i, l_i), \\ c_i \leq g_i(l_f, l_m, w_f, w_m, y), \\ 0 \leq T - l_i \leq H \quad \forall i = f, m, \end{cases} \quad (7)$$

where $g_i(l_f, l_m, w_f, w_m, y) \equiv \tilde{w}_i(T - l_i) + \tilde{y}_i$. The justification for preferring the sequential representation in this exercise is that it is computationally simpler, especially for the cases of absence of taxation and of individual taxation, described later in this section. The chosen functional form for the individual utility functions is:

$$U^i(c_i, l_i) = k_i \ln(c_i - \bar{c}) + (1 - k_i) \ln l_i, \quad (8)$$

where k_i denotes the marginal propensity to consume, and \bar{c} is a minimum level of consumption, assumed identical for all individuals. Without the introduction of \bar{c} , preferences would be homothetic, a restriction which is universally empirically rejected.

To describe the simulations precisely, we need to explain the generation of the variables and parameters needed to compute optimal allocations on the basis of equations (7) and (8), and to specify the budget constraints g_i . Parameter settings and variable generation are described in appendix A. In order to specify the budget constraint in (7), we first define the sharing rule for gross unearned income (equation (6)). We assume that gross unearned income is shared in proportion to gross wages:

$$\Upsilon(w_f, w_m, y) = \frac{y_f}{y} = \frac{w_f}{w_f + w_m}. \quad (9)$$

This choice is motivated, with drastic simplifications, by results from empirical studies (see *e.g.* Bourguignon *et al.*, 1993; Browning *et al.*, 1994). Refining the

sharing rule by letting it depend on actual unearned incomes would probably strengthen the distortions due to the use of the unitary representation, because of its implication of income pooling. However it turns out that the simple choice made here suffices to produce evidence of substantial distortions. Another refinement that would increase distortions would make the sharing rule explicitly dependent on the tax system. For example, Donni (2003) proposes to allow the sharing rule to depend on a set of variables describing the tax system (such as a set of the marginal tax rates). A similar approach is taken up in the REHO papers and in Beninger *et al.* (2007).

Next we turn to the tax system. In the absence of taxation (linear budget constraint), we have:

$$g_i(\cdot) = w_i(T - l_i) + y_i, \quad \forall i = f, m. \quad (10)$$

In the case of **personal** or individual taxation with a piecewise linear progressive tax schedule, i 's disposable income ($i = f, m$) is:

$$g_i^p(\cdot) = \sum_{k=1}^{s-1} (1 - \tau_p^k)(S_p^{k+1} - S_p^k) + (1 - \tau_p^s)(R_i^p - S_p^s), \quad (11)$$

where R_i^p is i 's gross income: $R_i^p = w_i \cdot T - l_i^p + y_i$. Vector $\tau_p = (\tau_p^1, \dots, \tau_p^{t_p})$ is the vector of marginal tax rates and $S_p = (S_p^1, \dots, S_p^{t_p})$ is the corresponding vector of tax brackets for individual taxation. There are t_p tax brackets and s represents the bracket for which $S_p^s \leq R_i^p < S_p^{s+1}$ if $s < t_p$ and $S_p^s \leq R_i^p$ if $s = t_p$. In that case, the assumptions made imply complete separation of the problems solved by the two spouses.

In the case of **common** or joint taxation with a piecewise linear progressive taxation rule, i 's disposable income is:

$$g_i^c(\cdot) = w_i(T - l_i^c) + y_i - I_i^c, \quad \forall i = f, m, \quad (12)$$

where I_i^c is i 's contribution to the total tax liability I^c of the household, where the latter is given by:

$$I^c(\cdot) = \sum_{k=1}^{s-1} \tau_c^k (S_c^{k+1} - S_c^k) + \tau_c^s (R^c - S_c^s), \quad (13)$$

where $R^c = \sum_i w_i \cdot (T - l_i^c) + y$ is household gross income. Notations for the tax brackets and marginal tax rates are similar to the case of personal taxation. We

assume that the tax liability of the household is shared proportionately to gross incomes, *i.e.* $I_i^c/I^c = R_i^c/R^c$. This assumption is arbitrary but it is the most convenient rule we could think of⁴.

The numerical values we choose for the tax parameters are: $\tau_p = \tau_c = (0, .3, .5)$, $S_p = (0, 5000, 10000)$ and $S_c = (0, 7800, 15000)$, so as to ensure revenue neutrality (approximately). All magnitudes pertain to a week, except the hourly wage rate. Numbers were chosen with French Francs in mind.

The collective model thus defined is perfectly deterministic. It is not clear to us whether or not this feature systematically puts the unitary model in a more disadvantageous situation than if some noise had been introduced in the model.

Simulated collective data sets

We have generated data for 2000 couples in three different cases: No taxation, joint taxation and individual taxation. Each individual is characterized by an observed heterogeneity factor which is the marginal propensity to consume, and has a budget constraint $g_i(l_f, l_m, w_f, w_m, y)$ which depends on the tax system and on the sharing rule Υ and has as arguments both leisure demands, both gross wage rates and household unearned income⁵. Given these functions, the optimal labor supply and consumption of each spouse is computed for each tax situation by solving (7) for each household, with real valued solutions, using Mathematica 4.0⁶. A summary of the results is given in table 1, and a more complete description is given in appendix B.

Table 1. Simulated data (means)

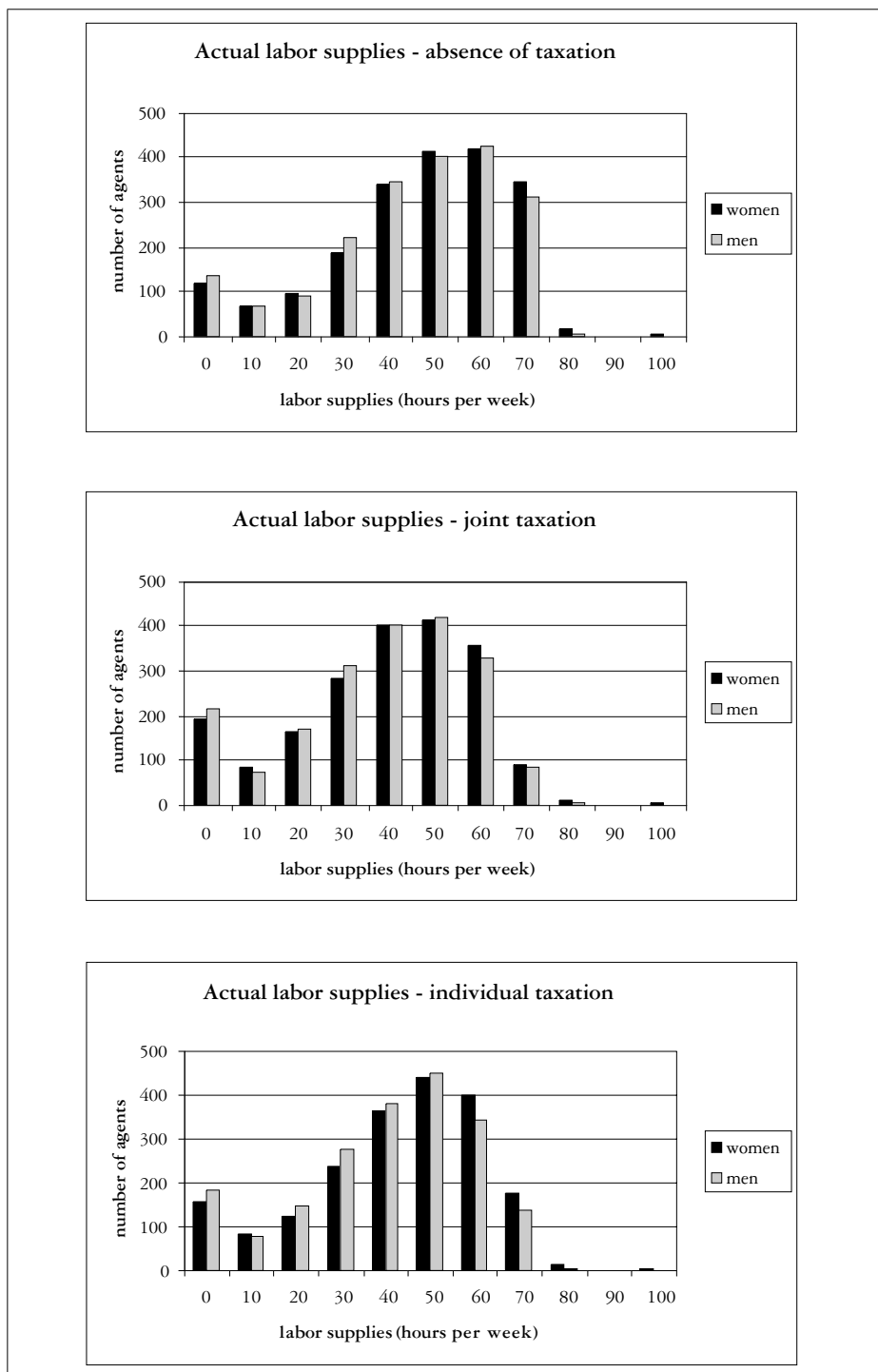
Variable	Symbol	No tax.	Joint tax.	Indiv. tax.
Her work hours	b_f	46.8	39.6	42.8
His work hours	b_m	45.8	38.6	40.7
Household cons.	c	13,730	10,695	11,026
Her wage	w_f		77.3	
His wage	w_m		102.3	
Unearned income	y		5,054	

⁴ Note that it is just as arbitrary (although realistic) to assume for the case of individual taxation that each spouse pays his own tax: A different sharing rule would be conceivable.

⁵ Since we have drawn the wages of husband and wife independently, there is no assortative matching in these data sets.

⁶ At this stage continuous labor supplies and consumptions are simulated.

Figure 1. Weekly hours of work



Regardless of the tax situation, the wife's labor supply, b_f , is on average a little higher than the husband's, b_m . This is not what is observed for childless couples in typical data sets, and a more complex specification of the sharing rule, or the specification of different preferences for males and females, would probably allow us to come up with simulated data more closely mimicking established stylized facts. But given the nature of this exercise, this is at most of minor importance. Note that here the only difference between husbands and wives is that men have on average higher wages, which translates into a higher share of unearned income, and thus into lower labor supply, given that leisure is a normal good in the preferences considered.

When the individuals are not taxed, women are more likely to prefer a part-time job (about 20 hours weekly), or to work overtime. Men are slightly more likely to work an "average" number of hours (30 to 40 weekly hours). But despite the wage differential, the distribution of labor supply is roughly the same for both sexes. Compared to real data for most countries, labor supply here is rather high, with a majority of individuals working between 30 and 70 hours per week. About 7% do not participate: These are essentially individuals with a high level of unearned income. Individuals with a high marginal propensity to consume are underrepresented among non participants.

The introduction of taxation has a large disincentive impact, but this varies with individual and household characteristics. Individuals with high wages and high unearned income are more affected, while the poorest are not affected at all.

In the specific case of the introduction of individual taxation, consider two identical women (same marginal propensity to consume, same wage and household unearned income), but assume that their husbands are endowed with different wages. The woman whose husband has the lower wage supports a larger disincentive effect than the other one, because of the specification of the income sharing rule.

In the case of joint taxation the disincentive effect is even larger, and again, better endowed individuals are more affected. The new aspect here is that, in case of a large wage gap between the spouses, the poorer one will also reduce his or her labor supply: A comparatively low wage rate implies inequality on two accounts within the household: As regards wage income, $w_i(T - l_i)$, and as regards the sharing of unearned income, y_i . Joint taxation is relatively unfavorable to the less well endowed spouse, as the latter bears part of the tax burden of the richer spouse. Again, individuals with a high marginal propensity to consume are more affected than others.

The fact that joint taxation has a larger disincentive effect than individual taxation does not result from differences in tax revenue: The tax brackets have been specified in such a way that tax revenues are virtually identical in both situations (see third section and appendix D). In fact, it turns out that joint taxation is more favorable for **households** than individual taxation: For the tax parameters used here and for identical gross income (labor supplies and unearned income fixed), a couple pays more tax in the case of individual taxation than in the case of joint taxation (about 25% more). However, joint taxation is relatively more favorable to the richer spouse (see sixth section, fourth subsection).

We now examine more specifically the impact of different variables and parameters on “collective” reactions to the introduction of one type of taxation or the other, and begin with household unearned income, y . As leisure is a normal good for each individual, leisure demand l_i ($i = f, m$) will increase with y . However, in the case of joint taxation there are other effects at work, because there is a degree of interdependence in the behavior of the spouses. It then becomes difficult to say *a priori* what the impact of a change in y will be.

The effect of a change in the wage rate w_i on labor supply b_i is even more ambiguous, as it depends on the other characteristics of the individual (marginal propensity to consume, spouse’s wage and household unearned income). Yet our simulated data show generally a negative association between wages and labor supply. This is due to the fact that the income effect of a change in the wage is reinforced by the corresponding change in the individual’s share of unearned income. In particular, individuals with the lowest wages often have a relatively high labor supply. Still, a clear effect is that an increase in w_i ($i = f, m$) has a negative impact on the spouse’s labor supply b_j ($j = f, m, j \neq i$), because of the specification of the sharing rule (9).

Of course, an increase in the marginal propensity to consume k_i induces individuals to work more.

The impact of changes in parameter values on c_i are less ambiguous. There is a positive impact of y , w_i and k_i on c_i . By contrast, w_j ($j = f, m, j \neq i$) has a negative impact through the sharing rule. And, almost needless to say, introducing taxation has a negative effect on consumption.

Econometric specification of unitary model

For the unitary model, we follow van Soest (1995) in specifying a direct translog household utility function, and adopt his discrete choice approach. The latter is well adapted to the estimation of preference parameters in the presence of nonlinear budget constraints. An alternative would be to specify the unitary model in such a way that it would correspond to specification (2) with a constant relative weight for the husband. Whether this would be in favour of the unitary model, in the sense of improving its capacity to track the collective data, is doubtful. Here we prefer to place ourselves in the situation of an econometrician who treats the data as if they were generated by a unitary model. In particular, this leads us to consider only aggregate consumption, which is typically available in survey data, rather than the individual consumptions available in our simulated collective data. A natural stance, given that our focus here is primarily empirical, rather than analytical – we want to point to the distortions entailed by estimation of a unitary model of the type generally used in empirical research when the data are generated by a collective model – is to specify a fairly flexible unitary model rather than constraining it *a priori* so that it is nested in our collective model. Estimating model (2) with constant λ would yield results in the unitary setting more directly comparable with the collective model, and thus lead to a better understanding of the sources of the discrepancies, but it would necessitate using information on individual

consumptions, which is typically missing in household surveys. We are not aware of any empirical study having estimated a household utility function as specified in (2).

Our specification of the household utility function thus takes the form:

$$\hat{U}(v) = v'Av + b'v, \quad (14)$$

where $v = (\ln c, \ln l_f, \ln l_m) \equiv (lc, ll_f, ll_m)$, and $\hat{U}(\cdot)$ represents the household's utility function. The 3×3 matrix $A = (\{\alpha_{ij}\})$, $i, j = c, f, m$, is symmetric, and $b = (\beta_c, \beta_f, \beta_m)$ is a vector of parameters. Subscripts c, f, m , are associated to lc, ll_f and ll_m , respectively. Consumption c results from the leisure choices through the budget constraint:

$$c = g(l_f, l_m, w_f, w_m, y).$$

We introduce only one type of observed heterogeneity, connected with the existence of three possible values for the marginal propensity to consume. This will take the form of two series of three dummies, one for the wife, and one for the husband (see appendix A), and we allow b to depend linearly on these observed characteristics.

We assume that each individual has $n = 11$ possible choices for his or her weekly labor supply: $b_i = T - l_i = 0, 10, \dots, 100$. This yields a set of $N = 121$ choices for the leisure demand pairs (l_f, l_m) of the spouses. If $\hat{U}(lc^j, ll_f^j, ll_m^j)$ denotes the utility generated by combination (c^j, l_f^j, l_m^j) , adding an error term ε_j , we define actual utility derived from combination j as:

$$\hat{U}_j = \hat{U}(lc^j, ll_f^j, ll_m^j) + \varepsilon_j \quad \forall j = 1, \dots, N. \quad (15)$$

Specifying the extreme value distribution for ε_j , defined by:

$$\Pr[\varepsilon_j < \varepsilon] = \exp(-\exp(-\varepsilon)), \quad (16)$$

leads to the multinomial logit model:

$$\Pr[\hat{U}_j > \hat{U}_k, \forall k \neq j] = \frac{\exp(\hat{U}(c^j, l_f^j, l_m^j))}{\sum_{k=1}^N \exp(\hat{U}(c^k, l_f^k, l_m^k))}. \quad (17)$$

Expression (17) represents the household's contribution to the likelihood.

In the absence of unobserved heterogeneity in the data, there would only be marginal returns to the estimation of more refined specifications allowing for such heterogeneity, such as some version of the mixed multinomial of McFadden and

Train (2000) or the heterogeneity augmented logit model of Chesher and Santos Silva (2002). Similarly, including fixed costs of work along the lines of Gong and van Soest (2002) would appear artificial, as no such costs are present in the data.

The following restrictions should be satisfied:

$$-\hat{U}_c^{-1} \begin{pmatrix} c_{l_f} & 1 & 0 \\ c_{l_m} & 0 & 1 \end{pmatrix} H\hat{U} \begin{pmatrix} c_{l_f} & c_{l_m} \\ 1 & 0 \\ 0 & 1 \end{pmatrix} > 0, \quad (18)$$

where $\hat{U}_x = \frac{\partial \hat{U}}{\partial x}$, $c_x = -\frac{\hat{U}_x}{\hat{U}_c}$, $H\hat{U} = \{\hat{U}_x, y\} = \left\{ \frac{\partial^2 \hat{U}}{\partial x \partial y} \right\}$ with $x, y = c, l_f, l_m$. This restriction is a consequence of the quasi-concavity of the household utility function and of the fact that \hat{U} is increasing in c (see. *e.g.* van Soest *et al.*, 1990). This last assumption is verified if:

$$2(\alpha_{cc}l_c + \alpha_{fc}ll_f + \alpha_{mc}ll_m) + \beta_c > 0. \quad (19)$$

Similar monotonicity restrictions apply w.r.t. the other arguments of \hat{U} .

Results

In this section we first give the estimation results for the unitary models, and the predictions of the labor supplies based on these estimates. We then perform the analysis of the different fiscal reforms, using the two types of models considered, and stress differences in positive and normative aspects of the corresponding evaluation.

Estimation of unitary models

Maximum likelihood estimates of the parameters of the utility function specified in the fifth section, and based on equation (17), are given in appendix C. Estimation was conducted in the three cases “absence of taxation”, “joint taxation” and “individual taxation”, yielding three vectors of estimates denoted $\hat{\theta}^o$, $\hat{\theta}^c$ and $\hat{\theta}^p$, respectively. The reason for performing the three estimations is the following. Since the individual underlying preferences are unchanged throughout the exercise, estimation results at the level of the household should in principle be fairly similar in all three situations. If we find that estimates for household preference parameters are affected by the form of the household budget constraint, this will result mainly from the misspecification embodied in the unitary representation of collective households⁷.

⁷ If we estimated the collective model on the three data sets, we could not expect to find identical estimates, but with the unitary model this sampling variation is compounded with the effect of misspecification.

Following van Soest, we interacted the dummies describing the marginal propensity to consume of each spouse only with that individual's leisure demand (see appendix A).

The estimates do differ significantly. Admittedly, a more thorough comparison of estimation results could focus on marginal effects combining the linear and quadratic coefficients. However, the differences we note are large enough for us to dispense with that further step.

The estimated coefficients $\hat{\beta}_i$, $i = c, l_f, l_m$, differ considerably depending on the data set used – and thus on the tax system. The coefficients of leisure demand are all significant in one case only, that of individual taxation. The signs of significant β coefficients for joint taxation agree with those for individual taxation. But all significant β coefficients for labor supply in the absence of taxation are positive, whereas they are negative for individual taxation. The coefficient $\hat{\beta}_c$ is strongly and significantly negative in all cases, but its magnitude doubles when going from absence of taxation to individual taxation, with the value for joint taxation inbetween. Note that, even if the linear impact of consumption on utility (when the household is not taxed) is strongly negative, the overall effect of consumption on household utility is positive for almost all observations: Condition (19) is satisfied, except for a few households where both spouses have a very large labor supply (connected with low wages and unearned income), especially in the case of individual taxation. The situation as regards labor supplies is less favorable, though.

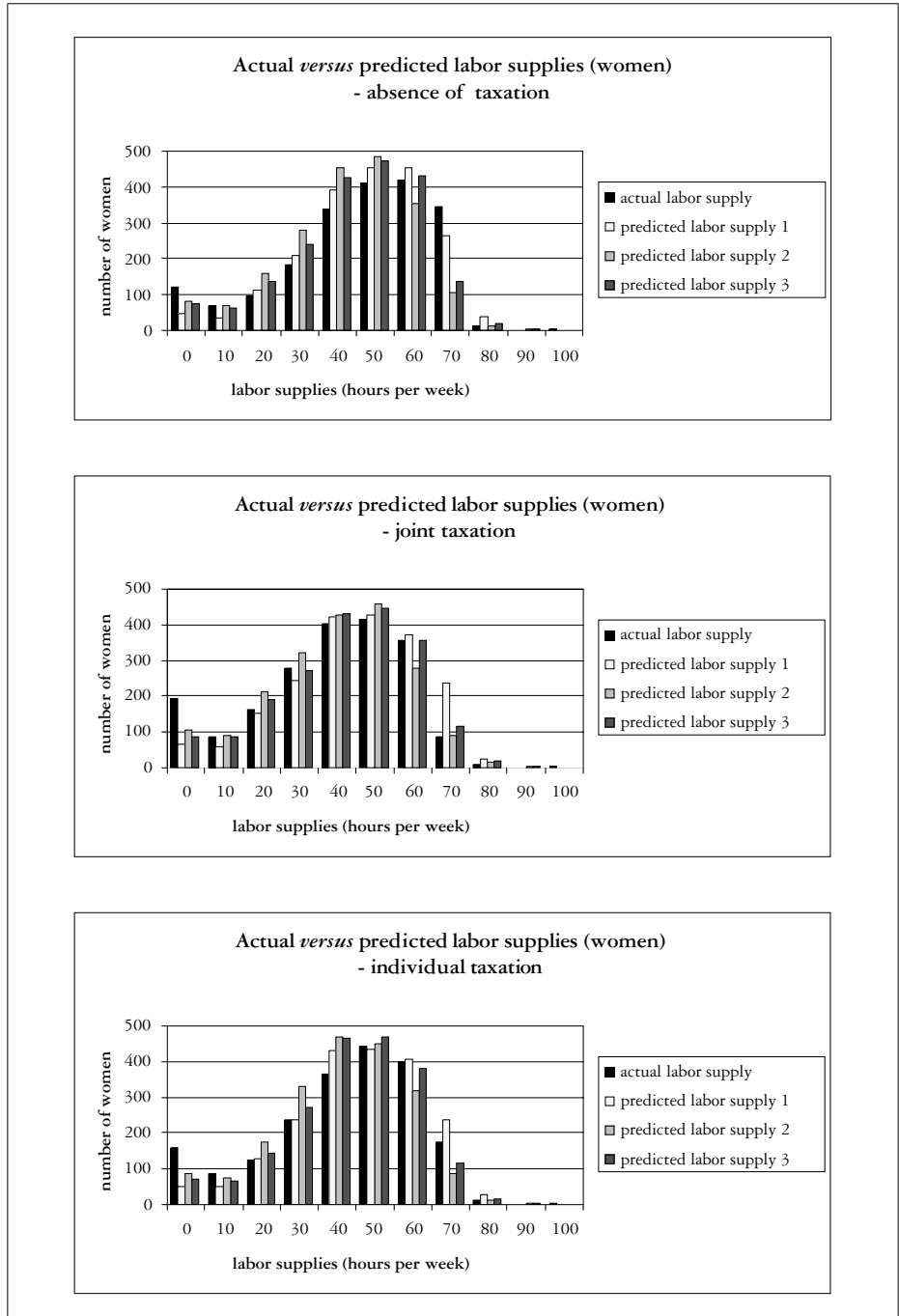
Quadratic effects, both direct and cross effects, all significant in all three estimations, are a little more stable w.r.t. the tax system, as no sign reversals appear here. The direct effects (coefficients α_{ii}) concerning leisure have similar magnitudes, although they are a little weaker in the case of individual taxation. Coefficient α_c is more sensitive to the tax system, and the same holds for cross effects (coefficients α_{ij} , with $i \neq j$), especially the cross effects consumption-leisure demand. Estimation results concerning the quadratic effects are fairly similar to those obtained with real data (see for example van Soest (1995), for Dutch data, and Wolf (1998), for German data). The restriction of quasi concavity of the utility function (18) is satisfied for most observations in each case.

Predictions with unitary models

These estimates can be used to make predictions of labor supplies, obtained by computing choice probabilities on the basis of equation (17) and taking the category with the highest choice probability as the chosen one. Note that, with three vectors of estimates and three datasets, we examine nine sets of predictions.

Predicted labor supplies have a lower dispersion than effective labor supplies, and a more symmetric distribution (see figure 2 for women; similar results are obtained for men; note that in the comparisons, we have discretized the collective labour supplies, in the same categories 0, 10, ..., 100 as for the predictions from the unitary models). In particular the number of nonworking individuals is underestimated in all cases and for all predictions. The estimation of household labor supply on the basis

Figure 2. Actual and predicted weekly hours of work



Note: Predicted labor supplies 1, 2 and 3 correspond to the use of parameter vectors $\hat{\theta}^a$, $\hat{\theta}^c$ and $\hat{\theta}^b$, respectively.

of real world data (see for instance van Soest, 1995) also indicates an overestimation of the participation rates. Van Soest argues that this phenomenon may result from the endogeneity of gross wage rates and from characteristics of the demand side of the labor market, and proposes a correction taking account of restrictions on hours. But here gross wages are perfectly exogenous and known for all agents, and labor supply is not constrained in any way. Thus, the overestimation of participation rates does not result from the neglect of institutional factors in the specification.

Predictions obtained with estimated coefficients $\hat{\theta}^c$ and $\hat{\theta}^p$ tend to underestimate labor supplies (except predictions using $\hat{\theta}^p$ in the case of joint taxation). The underestimation is larger when $\hat{\theta}^c$ is used. Predictions based on $\hat{\theta}^o$ overestimate part-time work and underestimate 'overtime'. Yet all vectors $\hat{\theta}$ are supposed to be consistent estimates of the same preference parameters, and given the fairly large sample size, predictions obtained with these vectors should not be too dissimilar.

The predictions look better when the coefficient vector used corresponds to the situation described (for example using $\hat{\theta}^c$ to predict labor supplies in the case of joint taxation), in the sense that more agents have identical predicted and actual labor supplies.

The differences shown in figure 2 may seem minor, but it should be stressed that these are only differences in the marginal distributions of hours worked. Even if these were identical, there could still be important differences at the individual level.

Analysis of fiscal reforms: Positive aspects

Tables 2 and 3 show that the adjustment of labor supplies following the introduction of taxation is also poorly predicted using the unitary model, especially in the case of joint taxation. For example, with the unitary model at least 110 women are predicted to offer unchanged numbers of hours, whereas they actually reduce their weekly labor supply by 10 hours or more after the introduction of joint taxation, and more women are predicted to keep their labor supply unchanged than is actually the case (see table 2). The reduction in hours offered after the introduction of any type of taxation is underestimated by the unitary models.

Tables 2 and 3 also show that predictions obtained with coefficients $\hat{\theta}^c$ and $\hat{\theta}^p$ are slightly worse than those obtained with $\hat{\theta}^o$, especially when $\hat{\theta}^p$ is used to predict the effect of introducing joint taxation and $\hat{\theta}^c$ is used for individual taxation.

Analysis of fiscal reforms: Normative aspects

We begin with the tax reform analysis based on the collective model. Figures 3.1 and 3.2 show the distribution of **individual** welfare gains, here negative, by decile of the distribution of gross income in the baseline situation without taxation (we show

Table 2. Effect of the introduction of joint taxation unitary *versus* collective model (women)

	≤ -30	-20	-10	0	Total
-10	3	43	406	143	595
0	10	106	718	562	1 396
10	0	4	0	5	9
Total	13	153	1124	710	2 000
1. using estimates $\hat{\theta}^o$					
	≤ -30	-20	-10	0	Total
≤ -10	3	49	381	111	545
0	10	96	738	591	1 435
≥ 10	0	8	5	7	20
Total	13	153	1 124	710	2 000
2. using estimates $\hat{\theta}^c$					
	≤ -30	-20	-10	0	Total
≤ -10	6	38	347	110	501
0	7	109	770	589	1 475
≥ 10	0	6	7	11	24
Total	13	153	1 124	710	2 000
3. using estimates $\hat{\theta}^p$					

Notes: Columns show the actual change in labor supply when joint (resp. individual) taxation is introduced, rows show corresponding predicted changes using the unitary model, in the three versions $\hat{\theta}^o$, $\hat{\theta}^c$ and $\hat{\theta}^p$. Diagonal in bold.

only figures for men, as the figures for women are very similar). As expected in the absence of redistribution of tax revenues, the introduction of taxation has a negative impact on welfare. Yet agents with low incomes do not incur losses when individual taxation is introduced, as they are not taxed at all. But some of them are affected by the introduction of joint taxation: Their spouse is rich enough for the couple to be taxed.

Figures 3.3 and 3.4 show the welfare consequences of the move from individual to joint taxation, for men and women. The fact that individual taxation is relatively more advantageous than joint taxation for low income individuals is confirmed. By contrast, high income agents are relatively better off with joint taxation, because it allows them to share the burden of taxation with the less well endowed spouse. Individual taxation is thus relatively more interesting for women, as their wage rates are on average 20% lower than men's.

Table 4 shows that in terms of welfare, 62% of the women gain in the move from joint to individual taxation, and some 20% are indifferent. The remaining 18% prefer joint taxation. By contrast, more men prefer joint taxation (about 36%), 17% are indifferent and the rest (47%) gains from individual taxation.

Table 3. Effect of the introduction of individual taxation unitary *versus* collective model (women)

	- 20	- 10	0	10	Total
- 10	19	209	184	2	414
0	44	473	1 010	0	1 527
10	2	3	54	0	59
Total	65	685	1 248	2	2 000
1. using estimates $\hat{\theta}^o$					
	- 20	- 10	0	10	Total
- 10	18	212	139	1	370
0	47	465	1 028	1	1 541
≥ 10	0	8	81	0	89
Total	65	685	1 248	2	2 000
2. using estimates $\hat{\theta}^c$					
	- 20	- 10	0	10	Total
- 10	19	205	117	2	343
0	45	468	1 000	0	1 513
≥ 10	1	12	131	0	144
Total	65	685	1 248	2	2 000
3. using estimates $\hat{\theta}^p$					

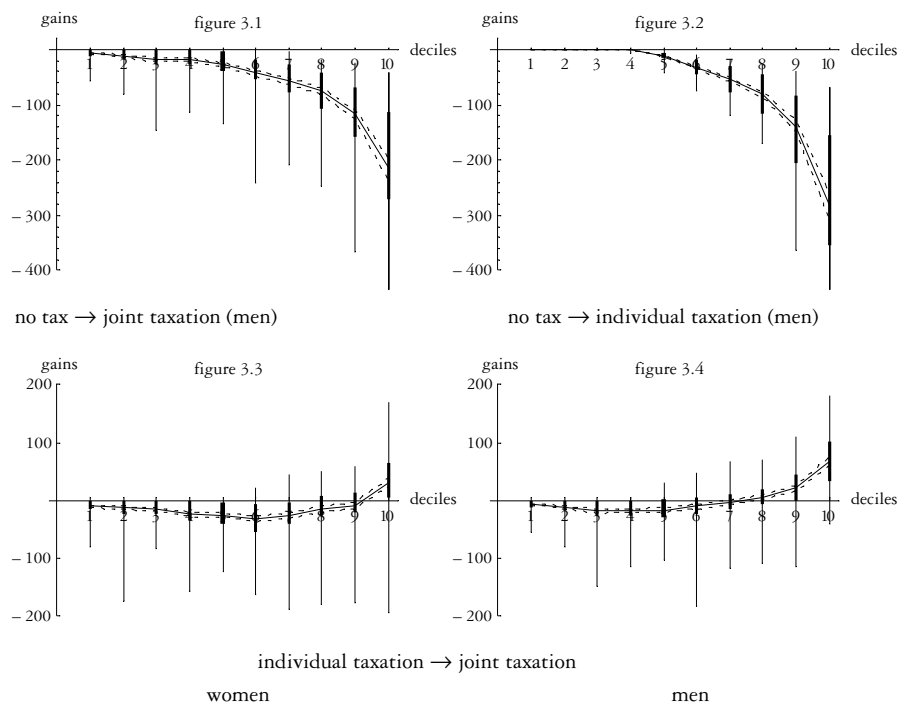
Notes: Columns show the actual change in labor supply when joint (resp. individual) taxation is introduced, rows show corresponding predicted changes using the unitary model, in the three versions $\hat{\theta}^o$, $\hat{\theta}^c$ and $\hat{\theta}^p$. Diagonal in bold.

There are relatively few couples for whom the direction of the relative advantage of each fiscal system is identical for both partners. This “conflictual” situation is essentially due to the wage rate differential and the ensuing difference in incomes: The richer partner finds joint taxation advantageous. For only 42% of the couples both partners fall in the same category (winner, loser, indifferent) in the switch between the two systems. These are partly households whose members are indifferent to each tax system anyway, as their incomes are too low to be taxed.

Descriptive statistics on the subsamples of winners and losers (not shown here in order to gain space) indicate that the marginal propensity to consume has an important impact on the welfare effects of the fiscal systems. Winners from joint taxation have on average a higher marginal propensity to consume. By contrast, household unearned income has no notable impact on the relative advantage of one system over the other in terms of the sign of welfare changes.

We now turn to the welfare impact of the reforms as described on the basis of the unitary models. Figures 4 show the distribution of **household** utility gains by decile of the distribution of gross household income in the baseline situation without taxation,

Figure 3. Gains in individual utility for different fiscal reforms collective model



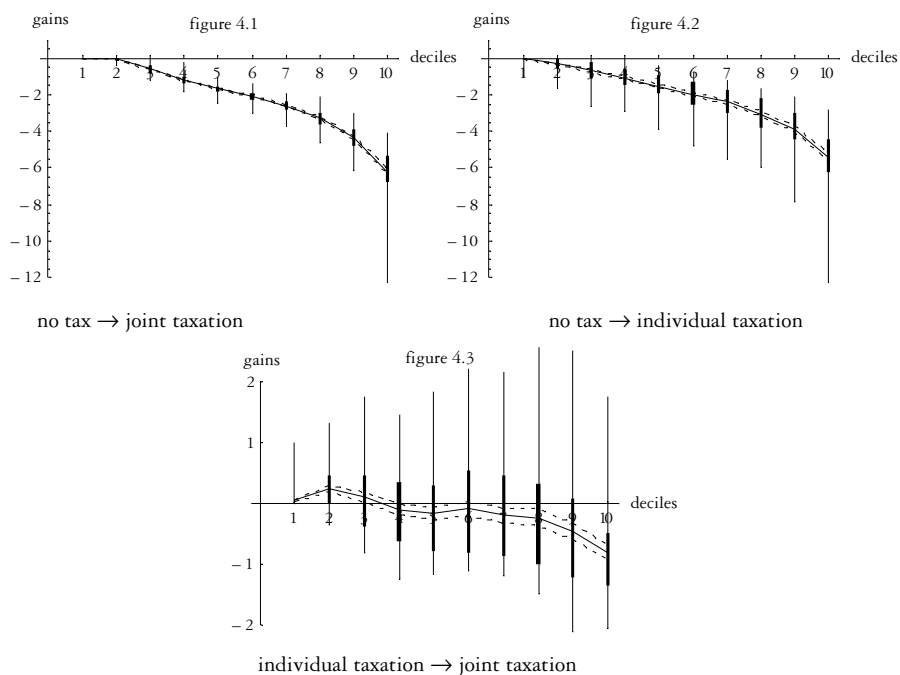
Notes: The figures represent mean utility gains by decile of the pre-reform gross individual income distribution for the introduction of joint taxation (Figure 3.1), individual taxation (Figure 3.2) and for the move from individual taxation to joint taxation (Figures 3.3 and 3.4), for women (Figure 3.3) and men (Figures 3.1, 3.2 and 3.4). The vertical lines show the range of gains, the rectangles show interquartile intervals, the solid line shows average gains, and the dotted lines show confidence intervals (\pm twice the standard error of the mean).

with utility gains computed from the unitary models⁸. Results concerning the introduction of taxation (joint or individual) are not in conflict with those obtained with the collective representation of the household. However, results apparently diverge as regards the move from individual to joint taxation: Whereas the richest **individuals** were mostly winners for the collective model, the majority of the richest **households** lose. This impression will be tempered when we look at individuals in households.

These results are described in table 4, which compares the qualitative welfare effects (gain, loss, indifference) of the move from individual to joint taxation, and of the inverse move, as predicted by both models. Note that both panels of table 4 are not exactly symmetrical: The number of households losing in the move from individual to joint taxation should in principle coincide with the number of winners in the reverse

⁸ The parameter used corresponds in each case to the initial tax system considered.

Figure 4. Household utility gains for different fiscal reforms, unitary model



Note: The figures represent household utility gains by decile of the distribution of pre-reform gross household income after the introduction of joint taxation (4.1), individual taxation (4.2) and in the move from individual to joint taxation (4.3).

move, whatever the estimate $\hat{\theta}$ used. This asymmetry results from the more pronounced underestimation of labor supplies with $\hat{\theta}^c$ than with $\hat{\theta}^p$. For about 100 households (5%), the predicted qualitative welfare effects are **contradictory**, in the sense that, in the collective framework, at least one household member gains in a reform and the partner does not lose, whereas using the unitary model the household loses or is indifferent. For the majority of these, both spouses are indifferent to the reform, whereas the household appears to lose in the unitary framework: The discrepancy results from the poor quality of unitary predictions of labor supply responses.

The “conflictual” situation uncovered by the collective model is masked by the unitary representation of household behavior (see also the simple example of Brett (1998), showing that Pareto improving reforms in a unitary setting are not necessarily Pareto improving in a collective setting). The relative advantage of a tax system in the collective model is essentially linked to the intra-family income differential, rather than only to total household income.

Table 4. Joint and individual taxation: Comparison of the collective and unitary models

	f_+			f_0			f_-			Total
	m_+	m_0	m_-	m_+	m_0	m_-	m_+	m_0	m_-	
hous ₊	5	33	217	51	49	1	377	2	3	738
hous ₀	1	1	0	4	216	2	2	2	11	239
hous ₋	1	2	141	2	18	16	286	18	539	1 023
Total	7	36	358	57	283	19	665	22	553	2 000
	f_-			f_0			f_+			Total
	m_-	m_0	m_+	m_-	m_0	m_+	m_-	m_0	m_+	
hous ₋	5	30	214	46	39	1	387	3	3	728
hous ₀	1	5	1	9	230	4	2	3	15	270
hous ₊	1	1	143	2	14	14	276	16	535	1 002
Total	7	36	358	57	283	19	665	22	553	2 000
Total f	(f_-)	401		(f_0)	359		(f_+)	1 240		2 000
Total m	(m_-)	729		(m_0)	341		(m_+)	930		2 000

Notes: The first panel corresponds to the move from individual to joint taxation, the second panel to the inverse move. Rows: winning (hous₊), indifferent (hous₀), and losing households (hous₋) on the basis of coefficients estimated in the departure situation, *i.e.* $\hat{\theta}^p$ for the first panel and $\hat{\theta}^c$ for the second. Columns: winning females (f_+) and males (m_+) – resp. f_- , m_- and f_0 , m_0 for losers or indifferent individuals, on the basis of the simulated collective data. Bold (italic) entries denote agreement (contradiction) between collective and unitary models.

Conclusion

On the basis of data simulated for 2000 collective households in three different fiscal situations, we have provided evidence on the distortions connected with the use of estimates from a flexible unitary model for the evaluation of fiscal reforms.

Firstly, the unitary model leads to fairly diverging estimates of household “preferences” depending on the budget constraint faced by the households; this is inasmuch problematic as these estimates are subsequently used as truly reflecting preferences in policy evaluation exercises.

Secondly, even using for each reform the unitary estimates corresponding to the departure situation, we find significant discrepancies in the prediction of positive effects of fiscal reforms (adjustment in the labor supplies of the spouses, and computation of tax revenues). In particular, the latter point shows that using a unitary representation may lead to non-negligible errors in the design of revenue neutral reforms.

Finally, the comparison of the evaluation of welfare effects of fiscal reforms with the collective and unitary models reveals substantial divergences, but also points to

the deep differences in their foundation itself. Using a unitary model leads the investigator to neglect resource allocation **within** the household, and this may mask the fact that a reform that seems advantageous for a majority of couples can in reality generate increased tensions and inequality within a substantial number of households. Conversely, using the collective model may lead to exaggerated emphasis on purely individual aspects.

An important development of this study, namely to generate data in a realistic way for different countries, taking account of the existing fiscal systems and reforms under discussion, as well as of the structure and the population, and to investigate how important the distortions illustrated here may be for six European countries, is documented in the REHO papers.

In future research, efforts should be invested in the estimation of collective models with non-participation and taxation. Papers going in this direction are Beninger (2007), Donni (2007) and Vermeulen (2006). A seemingly attractive path would be to resort to indirect inference (Gouriéroux *et al.*, 1993; Dridi and Renault, 2000; Dridi *et al.*, 2007), *i.e.* to formulate an auxiliary model, and to estimate the initial model by minimizing the discrepancy between estimates obtained from the auxiliary model on the original data on the one hand, and on data simulated with the initial model on the other hand. Theoretical results on the identification of collective models, or the lack of it, suggest that the simultaneous use of information from singles, and corresponding identifying assumptions, will be helpful.

References

- Apps P.F., Rees R. (1999). Individual *versus* joint taxation in models with household production, *Journal of Political Economy*, 107, pp. 393-403.
- Apps P.F., Rees R. (1997). Collective labor supply and household production, *Journal of Political Economy*, 105, pp. 178-190.
- Apps P.F., Rees R. (1996). Labor supply, household production and intra-family welfare distribution, *Journal of Public Economics*, 60, pp. 199-220.
- Apps P.F., Rees R. (1988). Taxation and the household, *Journal of Public Economics*, 35, pp. 355-369.
- Arrow K. (1951). *Choix collectif et préférences individuelles*, Paris, Calmann-Lévy.
- Bargain O., Beblo M., Beninger D., Blundell R., Carrasco R., Chiuri M.C., Laisney F., Lechene V., Longobardi E., Moreau N., Myck M., Ruiz-Castillo J. and Vermeulen F. (2006). Does the representation of household behavior matter for welfare analysis of tax-benefit policies? An introduction, *Review of Economics of the Household*, 4, pp. 96-112.

- Becker G.S. (1991). *A Treatise on the Family*, Cambridge (MA.), Harvard University Press.
- Beninger D. (2007). Would the WFTC help the German families to conciliate children and work? Empirical evidence based on a collective model, mimeo, ZEW Mannheim.
- Beninger D. (2003). Income taxation and the collective representation of the household: Theory and empirical evidence, Doctoral thesis, Université Louis Pasteur, Strasbourg I.
- Beninger D., Bargain O., Beblo M., Blundell R., Carrasco R., Chiuri M.C., Laisney F., Lechene V., Longobardi E., Moreau N., Myck M., Ruiz-Castillo J. and Vermeulen F. (2006). Tax reform analysis: The choice of the representation of household decision processes does matter, *Review of Economics of the Household*, 4, pp. 159-180.
- Beninger D., Laisney F. and Beblo M. (2007). Welfare analysis of fiscal reforms: Does the representation of the family decision process matter? Evidence for Germany, *Journal for Population Economics*, forthcoming.
- Ben Porath Y. (1982). Economics and the family-match or mismatch: A review of Becker's "A Treatise on the Family", *Journal of Economic Literature*, 20, pp. 52-64.
- Bergstrom T.C. (1989). A fresh look at the rotten kid theorem and other household mysteries, *Journal of Political Economy*, 97, pp. 1138-1159.
- Blundell R., Chiappori P.-A., Magnac T. and Meghir C. (1998). Collective labor supply: Heterogeneity and non-participation, Working Paper 20/98, Institute for Fiscal Studies, London.
- Bourguignon F. (1999). The cost of children: May the collective approach to household behavior help?, *Journal of Population Economics*, 12, pp. 503-521.
- Bourguignon F., Browning M., Chiappori P.-A. and Lechene V. (1993). Intra household of consumption: A model and some evidence from French data, *Annales d'Économie et de Statistique*, 29, pp. 137-156.
- Brett C. (1998). Tax reform and collective family decision-making, *Journal of Public Economics*, 70, pp. 425-440.
- Browning M., Bourguignon F., Chiappori P.-A. and Lechene V. (1994). Incomes and outcomes: A structural model of intra household allocations, *Journal of Political Economy*, 102, pp. 1067-1096.
- Browning M., Chiappori P.-A. (1998). Efficient intra household allocations: A general characterization and empirical tests, *Econometrica*, 66, pp. 1241-1278.
- Chesher A., Santos Silva J.M.C. (2002). Taste variation in discrete choice models, *Review of Economic Studies*, 69, pp. 147-168.

- Chiappori P.-A. (1997). Introducing household production in collective models labor supply, *Journal of Political Economy*, 105, pp. 191-208.
- Chiappori P.-A. (1992). Collective labor supply and welfare, *Journal of Political Economy*, 100, pp. 437-467.
- Chiappori P.-A. (1988). Rational household labor supply, *Econometrica*, 56, pp. 63-89.
- Chiuri M.C. (1999). Intra-household allocation of time and resources: Empirical evidence on a sample of Italian households with young children, *TMR Progress Report n° 5*, Tilburg.
- Deaton D.A., Muellbauer J. (1980). *Economics and Consumer Behavior*, Cambridge, Cambridge University Press.
- Donni O. (2007). Collective female labour supply: Theory and application, *Economic Journal*, 117, pp. 94-119.
- Donni O. (2003). Collective household labor supply: Non-participation and income taxation, *Journal of Public Economics*, 87 (5-6), pp. 1179-1198.
- Dridi R., Guay A. and Renault E. (2007). Indirect inference and calibration of dynamic stochastic general equilibrium models, *Journal of Econometrics*, 136, pp. 397-430.
- Dridi R., Renault E. (2000). Semi-parametric indirect inference, DP STICERD/ Econometrics EM/00/392, London School of Economics.
- Gong X., van Soest A. (2002). Family structure and female labour supply in Mexico City, *Journal of Human Resources*, 37, pp. 163-191.
- Gouriéroux C., Monfort A. and Renault E. (1993). Indirect inference, *Journal of Applied Econometrics*, 8, pp. 85-118.
- Haddad L., Kanbur R. (1990). How serious is neglect of intra household inequality, *Economic Journal*, 100, pp. 65-81.
- Hildenbrand W. (1998). How relevant are specifications of behavioral relations on the micro-level for modelling the time path of the population?, *European Economic Review*, 42, pp. 437-458.
- Hildenbrand W. (1994). *Market Demand: Theory and Empirical Evidence*, Princeton (N. J.), Princeton University Press.
- Lechene V. (1993). Représentation du ménage : essais en micro-économie théorique et appliquée, Thèse doctorale, EHESS, Paris.
- Lundberg S.J., Pollak R.A. and Wales T.J. (1996). Do husbands and wives pool their resources? Evidence from the United Kingdom child benefit, *Journal of Human Resources*, 32, pp. 463-480.

- McDonald J.B. (1984). Some generalized functions for the size distribution of income, *Econometrica*, 52, pp. 647-663.
- McFadden D., Train K. (2000). Mixed MNL models for discrete response, *Journal of Applied Econometrics*, 15, pp. 447-470.
- Myck M., Bargain O., Beblo M., Beninger D., Blundell R., Carrasco R., Chiuri M.C., Laisney F., Lechene V., Longobardi E., Moreau N., Ruiz-Castillo J. and Vermeulen F. (2006). Tax reform analysis using collective models of household labor supply, *Review of Economics of the Household*, 4, pp. 129-158.
- Samuelson P.A. (1956). Social indifference curves, *Quarterly Journal of Economics*, 70, pp. 1-22.
- Sen A.K. (1984). Economics and the Family, *in: Resources, Values and Development*, Cambridge (MA.) and London, Harvard University Press.
- van Soest A. (1995). Structural models of family labor supply: A discrete choice approach, *Journal of Human Resources*, 30, pp. 63-88.
- van Soest A., Woittiez I. and Kapteyn A. (1990). Labor supply, income taxes and hours restrictions in the Netherlands, *Journal of Human Resources*, 25 (3), pp. 517-558.
- Vermeulen F. (2006). A collective model for female labor supply with non-participation and taxation, *Journal of Population Economics*, 19, pp. 99-118.
- Vermeulen F. (2002). Collective household models: Principles and main results, *Journal of Economic Surveys*, 16, pp. 533-564.
- Vermeulen F., Bargain O., Beblo M., Blundell R., Carrasco R., Chiuri M.C., Laisney F., Lechene V., Moreau N., Myck M. and Ruiz-Castillo J. (2006). Collective models of household labor supply with non convex budget sets and non-participation: A calibration approach, *Review of Economics of the Household*, 4, pp. 113-127.
- Wolf E. (1998). Do hours restrictions matter? A discrete family labor supply model with endogenous wages and hours restrictions, ZEW Discussion Paper 98-44.

APPENDIX A

Parameters and variables

The parameter values are common to all individuals (like total available time, for instance), the variables are particular to each individual (like gross wage rates). In the latter case, the individual values result from i.i.d. sampling for each individual (for wage rates and for the marginal propensities to consume) and for each couple (unearned income).

Parameters

The fixed parameters, *i.e.* minimum consumption, \bar{c} , total time available, T , and maximal working time, H , are common to the whole population. All pertain to a week. Minimum consumption per week is set to $\bar{c} = 200$ monetary units (we had French Francs per week in mind when setting monetary values, but the actual units really are irrelevant). Total time available, in hours, is the duration of a week *minus* the time devoted to biological needs (sleeping, eating, etc.): $T = 120$ hours. This corresponds to the upper bound for leisure. The maximum duration of work is set to $H = 100$ hours. Note that we do not consider any domestic production in this model.

Variables

As mentioned above, these are the wage rates, unearned incomes and marginal propensities to consume.

Wage distributions are highly skewed to the right, and extremely low wages are rare in the rich countries (see *e.g.* Hildenbrand, 1994 and 1998). With these characteristics in mind, we specify the following density for the wages – this is a special case of the Fisk distribution (see *e.g.* McDonald, 1984):

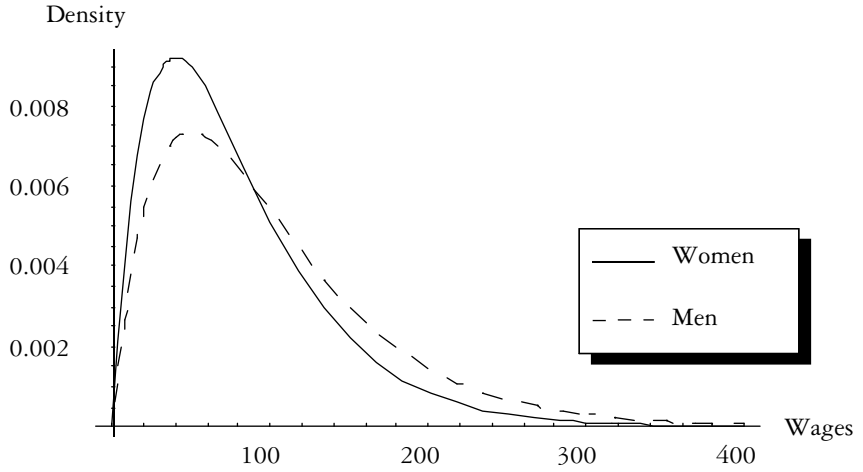
$$f_{w_i}(w_i) = \frac{w_i}{d_i^2} \exp\left(-\frac{w_i}{d_i}\right) \mathbf{1}(w_i \geq 0) \quad \forall i = f, m. \quad (20)$$

The corresponding cumulative probability function is:

$$\begin{aligned} F_{w_i}(w_i) &= \mathbf{1}(w_i \geq 0) \int_0^{w_i} \frac{t}{d_i^2} \exp\left(-\frac{t}{d_i}\right) dt \\ &= \mathbf{1}(w_i \geq 0) \left[1 - \left(\frac{w_i}{d_i} + 1 \right) \exp\left(-\frac{w_i}{d_i}\right) \right] \end{aligned} \quad (21)$$

where $d_f = 40$ and $d_m = 50$. Expectation, standard error and mode are respectively $E(w_i) = 2d_i$, $\sigma(w_i) = d_i\sqrt{2}$, and mode $(w_i) = d_i$. Figure A1 depicts the resulting wage distributions.

Figure A1. Wage densities for men and women



For **unearned income** we assume the exponential distribution:

$$f_y(y) = \frac{1}{\theta} \exp\left(-\frac{y}{\theta}\right) \mathbf{1}(y \geq 0), \quad (22)$$

with $\theta = 5000$. Expectation, standard error and mode are $E(y) = \sigma(y) = \theta$, and mode $(y) = 0$

Individuals are assumed to have either a low marginal propensity to consume ($k_i = .4$), or a medium one ($k_i = .5$) or a high one ($k_i = .6$). Define β as a realization of the Beta distribution $B(2.6, 2.6)$. We set:

$$\begin{aligned} k_i &= .4 & \text{if } \beta \leq .4 \\ k_i &= .5 & \text{if } .4 < \beta < .5 \\ k_i &= .6 & \text{if } \beta \geq .6 \end{aligned}$$

These choices result in about a third of observations in each category, and $E(k_i) = \text{mode}(k_i) = .5$, and $\sigma(k_i) = .08$. The dummies $\mathbf{1}_{i1}$, $\mathbf{1}_{i2}$, and $\mathbf{1}_{i3}$ take value 1 if $k_i = .4$, $.5$ or $.6$, respectively, and 0 otherwise.

APPENDIX B

Simulated data

Table B1. Simulation results

Variable	Mean	s.d.	min.	max.
Exogenous variables				
w_f	77.3	55.0	2.1	425.3
w_m	102.3	71.3	2.8	440.8
y_f	2,205	2,711	1	20,627
y_m	2,850	3,706	1	52,560
y	5,055	5,460	2	56,310
Choice variables, no taxation				
c_f	5,972	4,115	218	29,931
c_m	7,758	5,331	291	52,567
c	13,730	6,404	1,863	56,310
b_f	46.8	19.7	0	100
b_m	45.8	19.6	0	89
l_f	73.2	19.7	20	120
l_m	74.2	19.6	31	120
Choice variables, joint tax.				
c_f	4,693	2,856	218	18,134
c_m	6,001	3,474	281	31,829
c	10,694	3,665	1,863	34,095
b_f	39.6	19.6	0	100
b_m	38.6	19.4	0	89
l_f	80.4	19.6	31	120
l_m	81.4	19.4	20	120
Choice variables, ind. tax.				
c_f	4,977	2,684	218	17,106
c_m	6,049	3,132	291	29,784
c	11,026	3,906	1,863	33,527
b_f	42.8	19.8	0	100
b_m	40.7	19.6	0	89
l_f	77.2	19.8	20	120
l_m	79.3	19.6	31	120

APPENDIX C

Estimation of unitary model

Table C1. Estimation results – no taxation coefficient vector $\hat{\theta}^o$

Parameter	Variable	Coef.	s.e.	t
$\beta_{f\iota_{f1}}$	$ll_f \times \iota_{f1}$	23.22	5.72	4.1
$\beta_{f\iota_{f2}}$	$ll_f \times \iota_{f2}$	15.56	5.69	2.7
$\beta_{f\iota_{f3}}$	$ll_f \times \iota_{f3}$	7.24	5.61	1.3
$\beta_{m\iota_{m1}}$	$ll_m \times \iota_{m1}$	14.30	6.58	2.2
$\beta_{m\iota_{m2}}$	$ll_m \times \iota_{m2}$	5.15	6.53	.8
$\beta_{m\iota_{m3}}$	$ll_m \times \iota_{m3}$	-3.40	6.49	-.5
β_c	lc	-49.92	10.76	-4.7
α_{ff}	$ll_f \times ll_f$	-17.59	.57	-31.0
α_{mm}	$ll_m \times ll_m$	-18.04	.61	-29.8
α_{cc}	$lc \times lc$	1.51	.42	3.6
α_{fm}	$ll_f \times ll_m$	25.39	.81	31.4
α_{fc}	$ll_f \times lc$	3.45	.44	7.9
α_{mc}	$ll_m \times lc$	5.23	.52	10.1

Table C2. Estimation results – joint taxation coefficient vector $\hat{\theta}^c$

Parameter	Variable	Coef.	s.e.	t
$\beta_{f\iota_{f1}}$	$ll_f \times \iota_{f1}$	-13.42	4.60	-2.9
$\beta_{f\iota_{f2}}$	$ll_f \times \iota_{f2}$	-20.80	4.59	-4.5
$\beta_{f\iota_{f3}}$	$ll_f \times \iota_{f3}$	-28.86	4.50	-6.4
$\beta_{m\iota_{m1}}$	$ll_m \times \iota_{m1}$	4.45	5.26	.8
$\beta_{m\iota_{m2}}$	$ll_m \times \iota_{m2}$	-4.18	5.19	-.8
$\beta_{m\iota_{m3}}$	$ll_m \times \iota_{m3}$	-12.47	5.13	-2.4
β_c	lc	-68.71	2.45	-28.0
α_{ff}	$ll_f \times ll_f$	-19.02	.64	-29.5
α_{mm}	$ll_m \times ll_m$	-20.17	.70	-28.8
α_{cc}	$lc \times lc$	1.64	.07	25.1
α_{fm}	$ll_f \times ll_m$	30.80	.96	32.1
α_{fc}	$ll_f \times lc$	6.39	.33	19.6
α_{mc}	$ll_m \times lc$	5.95	.37	15.9

Table C3. Estimation results – individual taxation coefficient vector $\hat{\theta}^p$

Parameter	Variable	Coef.	s.e.	t
$\beta_{f\iota_{f1}}$	$ll_f \times \iota_{f1}$	-17.97	4.79	-3.7
$\beta_{f\iota_{f2}}$	$ll_f \times \iota_{f2}$	-24.59	4.81	-5.1
$\beta_{f\iota_{f3}}$	$ll_f \times \iota_{f3}$	-31.44	4.77	-6.6
$\beta_{m\iota_{m1}}$	$ll_m \times \iota_{m1}$	-21.06	5.32	-4.0
$\beta_{m\iota_{m2}}$	$ll_m \times \iota_{m2}$	-28.93	5.31	-5.4
$\beta_{m\iota_{m3}}$	$ll_m \times \iota_{m3}$	-36.50	5.30	-6.9
β_c	lc	-91.34	3.50	-26.1
α_{ff}	$ll_f \times ll_f$	-13.80	.52	-26.6
α_{mm}	$ll_m \times ll_m$	-15.01	.58	-25.7
α_{cc}	$lc \times lc$	2.35	.09	26.5
α_{fm}	$ll_f \times ll_m$	21.23	.73	29.2
α_{fc}	$ll_f \times lc$	6.22	.44	14.1
α_{mc}	$ll_m \times lc$	8.14	.49	16.6

APPENDIX D

Tax revenue

The first column of table D shows the amount of tax revenue in the situations of joint and individual taxation, computed from the simulated data. The small discrepancy between both figures could of course be eliminated by adjusting, say, the tax brackets in the joint taxation situation. However the subsequent columns show that this is of minor importance. For each estimated parameter vector and for each fiscal system, we report the tax revenue associated with the predicted labor market situation of each household. The misspecification associated with the unitary representation leads to important discrepancies. In all cases the unitary model predicts substantially larger tax revenues for joint taxation, whereas the actual tax revenue is actually marginally lower than for individual taxation. This point is important, because the evaluation of fiscal reforms often entails the specification of revenue neutral reforms.

The fact that the worst predictions are obtained using $\hat{\theta}^o$ is not surprising, since the introduction of taxation leads to large adjustments in labor supplies. However it is slightly puzzling to find that the revenue for individual tax is better predicted using $\hat{\theta}^c$ than $\hat{\theta}^p$. No such reversal appears for joint taxation. Another puzzle lies in the fact that tax revenues predicted using $\hat{\theta}^c$ and $\hat{\theta}^p$ are larger than the actual tax revenue, since the unitary model leads to an under-estimation of labor supplies (see sixth section, subsection 2). In fact, low levels of labor supply are rather over-estimated and notably the number of participants is overstated. But individuals with low labor supply have on average higher unearned income and are thus more likely to be taxed.

Table D. Tax revenues

Model tax system	Collective	Unitary		
		$\hat{\theta}^o$	$\hat{\theta}^c$	$\hat{\theta}^p$
Joint taxation	2,999,920	4,096,180	3,289,330	3,490,220
Individual taxation	3,026,510	3,801,910	3,121,990	3,324,170
Variation	-.88%	+ 7.74%	+ 5.36%	+ 5.00%